

A discusión

ON THE INCOME VELOCITY OF MONEY IN A CASH-IN-ADVANCE ECONOMY WITH CAPITAL*

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ABSTRACT

A stochastic growth model with money introduced via a cash-in-advance constraint is used to analyze the behaviour of the income velocity of real monetary balances. Agents can purchase consumption goods only using government issued money and capital is a credit good. The cash-in-advance constraint may become nonbinding because of the uncertainty about the realization of the state of the economy. Changes in the income velocity of money due to a precautionary money demand are studied. We find that despite the precautionary money demand does not introduce significant changes into the volatility of the income velocity, its presence can alter the relationship between the growth rate of money supply and the income velocity.

JEL classification numbers: E40

Key words: Cash-in-advance; Income velocity; Precautionary money demand.

1 Introduction

In this paper we study the changes in the income velocity of money that arise due to a precautionary money demand. Qualitative analysis of this effect in a model without capital was performed in Svensson (1985) and in a model with capital in Hromcová (1998). Here we evaluate the changes quantitatively in a model with capital.

There are several effects that may contribute to the variability of the income velocity of money. One is the reaction of real balances to the changes in the nominal interest rates.¹ When the nominal interest rate increases, it is less attractive to hold money and agents look for some alternative means of payment. This fact is captured in the literature by allowing for 'cash' and 'credit' goods, as for example in Lucas (1984), Lucas and Stokey (1983, 1987) and Jones and Manuelli (1995).² In such cash-credit models agents have a possibility of substituting between two ways of acquiring consumption goods. Therefore, when the opportunity cost of holding money increases, agents can switch from cash consumption towards credit consumption. This affects the real balances, and the income velocity of money varies accordingly.

Other types of models introduce changes into the money demand exogenously, via a modified cash-in-advance constraint, see for example Canzoneri and Dellas (1998), Col-lard, Dellas and Ertz (2000) or Caballé and Hromcová (2001). In these models agents are allowed to exchange a fraction of the current period income for consumption without using money. When this fraction increases, individuals economize on their real balances holdings and this implies a change in the money velocity.

As Lucas (1988) shows, standard cash-in-advance models can explain systematic changes in velocity due to income and interest rates, but not due to other factors. Trends in velocity are attributed to financial innovations and technological improvements in the financial sector and to the fact that the monetary policy influences the decisions of agents to devote resources to the creation of money substitutes (see for example Cole and Ohanian (2002) or Gordon, Leeper and Zha (1998) for surveys on papers which deal with these topics).

Another approach to model variable income velocity of money can be found in Svensson (1985). Letting the goods market open before the financial exchange takes place, agents are forced to make their decisions on the money holdings before the state of the economy is known. Such information structure may lead to a precautionary money demand in some states of the world, since agents may end up holding more money than they actually need in order to purchase consumption goods. In this way their cash-in-advance constraint may become nonbinding. Therefore, uncertainty might be a source of fluctuations of the money velocity. Calibrating the model of Svensson (1985) one finds that in states of the world when cash-in-advance constraint becomes nonbinding velocity drops below unity, but the quantitative changes are negligible. Hodrick, Kocherlakota and Lucas (1991) evaluate the

¹Plotting velocity and interest rates, as done for example in McGrattan (1998a), it can be seen that these two variables exhibit a similar pattern.

²In the words of Lucas (1994), 'cash' goods are consumed using money and 'credit' goods are exchanged directly for agents savings or are acquired issuing private securities.

stochastic properties of endogenous variables in models which do not necessarily restrict the cash-in-advance constraint to be binding in all states. They analyze cash-only and cash-credit models without capital. In their model, the choice between cash and credit goods depends on the preference parameters, particularly on the relative weight the agent gives to credit goods. If the weight given to the credit good is zero, the model reduces to a model of Svensson (1985) which exhibits small variability. They conclude that velocity varies in the cash-credit model because agents substitute between cash and credit goods, and the cash-in-advance constraint for the cash good almost always binds.

This paper extends the analysis of the income velocity of real balances done in Svensson (1985) to a model with capital. We concentrate on the variability of the income velocity of money that arises only due to the precautionary money demand and disregard other effects that may influence changes in the money velocity. We want to answer if the precautionary money demand may be considered as a significant mechanism to explain some part of money velocity fluctuations in a more general model. We do not analyze stochastic properties of other endogenous variables. The analysis of business cycle properties in a cash-in-advance economy with capital is done for example in Cooley and Hansen (1989).

We introduce capital into the model of Svensson (1985). To perform the analysis we allow for endogenous growth accruing from a linear (AK) technology. Capital will be taken as a credit good.³ This specification of the model can deliver realistic values for money velocity in equilibrium.⁴ Depending on the growth rate of money supply and the technology shock realization, agents might make a precautionary money demand in some periods. We want to characterize under which conditions a precautionary money demand occurs, and to evaluate how much variability is introduced by a precautionary money demand into the income velocity of money.

The question addressed in this work is in some way similar to the one of Hodrick, Kocherlakota and Lucas (1991). Hodrick, Kocherlakota and Lucas (1991) evaluate stochastic properties of endogenous variables for a set of chosen parameters, paying close attention to the volatility of the income velocity of money. The focus of the present work lies in looking for circumstances in which the cash-in-advance constraint switches from binding to nonbinding due to the state of the world and evaluating quantitative changes precautionary money demand introduces into the behavior of the income velocity. As we consider a more general framework, a model with capital, average levels of velocity similar to the ones found in the data can be generated.

The model does not admit a closed form solution and has to be solved numerically. We apply the method of parameterized expectations described in Den Haan and Marcet (1990). The reason why we choose this technique is that the algorithm allows to work with nonbinding constraints without any additional difficulty.

Our results suggest that the presence of the precautionary money demand alters some equilibrium properties. We find that our model exhibits Tobin effect when the cash-in-

³In the literature which deals with cash-credit goods, for example Lucas (1984), Lucas and Stokey (1983, 1987), Ireland (1994), credit goods are represented as a class of consumption goods. Nevertheless, it seems natural to consider capital as a broad class of credit good. It does not provide utility, but its accumulation yields to growth.

⁴A neoclassical technology would allow us to work with a model with a steady state and simplify the equilibrium analysis, but the steady state velocity level would not lead to realistic values.

advance constraint is always binding or always nonbinding. When the cash-in-advance constraint changes from binding to nonbinding according to the state of the world, inflation acts as a tax on capital. The relationship between the growth rate of money supply (inflation) and the income velocity changes accordingly. Concerning the income velocity of money, precautionary money demand affects significantly the magnitude of its fluctuations when the cash-in-advance constraint appears to be nonbinding in all states of the world. We also find that capital acts like credit good in Hodrick, Kocherlakota and Lucas (1991), and the model can deliver reasonable velocity fluctuations when the cash-in-advance constraint always binds.

The remainder of the paper is organized as follows. The model is described in section 2. In section 3 we describe the numerical algorithm and we calibrate the model to the US economy. In section 4 we analyze the equilibrium behavior of the economy. In particular, we study the changes in the income velocity of money caused by a precautionary money demand. Final conclusions are summarized in section 5.

2 Model Description

2.1 The Household Problem

We consider a representative agent economy. There is one consumption good which must be paid using currency. The economy has an infinitely lived representative household with preferences represented by the utility function

$$E_t \left\{ \sum_{j=t}^{\infty} \beta^{j-t} \left(\frac{c_j^{1-\theta} - 1}{1-\theta} \right) \right\} \quad (1)$$

where c_j is the consumption in period j and $\theta > 0$ can be viewed either as the inverse of the elasticity of intertemporal substitution or as the index of relative risk aversion.

The production and trade is characterized like in Lucas and Stokey (1983). Each household is composed of a worker-shopper pair. The timing of the events is the following. Agents enter the period t with a certain amount of monetary balances M_t and bonds B_t , carried over from the previous period, and the capital stock k_t , which is the result of the previous periods investment. A representative worker produces via the net production function

$$y_t = A_t k_t \quad (2)$$

where A_t is the shock to technology.⁵ We assume that the logarithm of technology shocks follows an autoregressive process,

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_{A,t}, \quad (3)$$

⁵To justify the constant return assumption we typically interpret the capital stock as a broad measure that may include also human capital. The class of models that reduce to an AK model are discussed in chapter 4 of Barro and Sala-i-Martin (1995). See also McGrattan (1998b) for the defence of AK models in growth theory.

where $\bar{A} > 0$ is the steady state value of the technology, $0 < \rho_A < 1$ and $\varepsilon_{A,t}$ is white noise, $\varepsilon_{A,t} \sim N(0, \sigma_A^2)$. Capital depreciates at a rate δ . The investment evolves as

$$i_t = k_{t+1} - (1 - \delta)k_t.$$

Prior to any trading agents learn the state of the economy A_t . The production takes place and y_t is realized. The worker stays at the place of production during the whole period. Goods market opens and consumption takes place. The shopper visits various stores to acquire consumption goods carrying the monetary balances of the household. Purchases are subject to the liquidity constraint

$$c_t \leq \frac{M_t}{p_t}. \quad (4)$$

The part of the production which is not consumed is invested into capital. Next, the monetary transfer $X_t = (\mu - 1)M_t$ from the government takes place. We denote μ as the growth rate of money supply which is set constant for all time periods. At the end of the period asset market opens. Agents can purchase one-period pure discount bonds paying B_{t+1} units of money at period t that pay $R_{t+1}B_{t+1}$ units of money when they mature in $t+1$. Therefore, R_{t+1} is the gross nominal interest rate between period t and $t+1$. Bonds are in zero net supply. Monetary balances M_{t+1} to be carried over to the next period are chosen. The budget constraint that agents are facing is thus

$$c_t + k_{t+1} - (1 - \delta)k_t + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{p_t} \leq A_t k_t + \frac{M_t}{p_t} + \frac{R_t B_t}{p_t} + \frac{X_t}{p_t}. \quad (5)$$

The real wealth of a household in period t is in the right hand side of (5) and consists of the current period output, real monetary balances, real return on bonds and the real lump sum transfer from the government. Current period wealth is used to consume, to invest into capital, and to save in the form of money and bonds, as can be seen in the left hand side of (5).

Definition: Given the set of initial conditions M_1, k_1, B_1 and the growth rate of the money supply μ the equilibrium consists of stochastic processes $\{c_t, M_{t+1}, k_{t+1}, B_{t+1}, R_{t+1}, p_t\}_{t=1}^{\infty}$ such that

- (a) a representative household chooses the stochastic sequences $\{c_t, M_{t+1}, k_{t+1}, B_{t+1}\}_{t=1}^{\infty}$ in order to maximize the expected discounted utility (1) subject to the budget constraint (5) and the cash-in-advance constraint (4),
- (b) markets for goods, money and bonds clear in every period,

$$c_t + k_{t+1} - (1 - \delta)k_t = A_t k_t, \quad (6)$$

$$M_{t+1} = M_t + X_t = \mu M_t, \quad (7)$$

$$B_{t+1} = 0. \quad (8)$$

3 Model Transformation and Numerical Solution

3.1 Model Transformation

Since we deal here with an endogenous growth model, variables are not stationary. We will work with variables in ratios. We define \hat{c}_t as the ratio of consumption and current period capital and γ_t as the gross rate of growth of capital,

$$\hat{c}_t = \frac{c_t}{k_t} \text{ and } \gamma_t = \frac{k_{t+1}}{k_t}.$$

Then, we define \hat{m}_t as stationary real monetary balances, i.e. the ratio of real balances and the current period capital

$$\hat{m}_t = \frac{\frac{M_t}{p_t}}{k_t}.$$

Let $\hat{\lambda}_t$ and $\hat{\eta}_t$ be stationary Lagrange multipliers associated with the budget constraint (5) and the cash-in-advance constraint (4).

The system of equations that characterize the equilibrium, are the first order conditions

$$\hat{c}_t^{-\theta} = \hat{\lambda}_t + \hat{\eta}_t, \quad (9)$$

$$\hat{\lambda}_t \hat{m}_t = \frac{\beta}{\mu} E_t \left[\left(\hat{\lambda}_{t+1} + \hat{\eta}_{t+1} \right) \hat{m}_{t+1} \gamma_t^{1-\theta} \right], \quad (10)$$

$$\hat{\lambda}_t = \beta E_t \left[\hat{\lambda}_{t+1} (A_{t+1} + 1 - \delta) \gamma_t^{-\theta} \right], \quad (11)$$

$$\hat{\lambda}_t \hat{m}_t = \frac{\beta}{\mu} R_{t+1} E_t \left[\hat{\lambda}_{t+1} \hat{m}_{t+1} \gamma_t^{1-\theta} \right], \quad (12)$$

the cash-in-advance constraint

$$\hat{\eta}_t \geq 0 \text{ and } \hat{\eta}_t (\hat{m}_t - \hat{c}_t) = 0, \quad (13)$$

and the equilibria in goods, money and bonds markets

$$\hat{c}_t + \gamma_t - (1 - \delta) = A_t, \quad (14)$$

$$M_{t+1} = \mu M_t, \quad (15)$$

$$B_{t+1} = 0. \quad (16)$$

Comparing (10) and (12) we get an expression for the nominal interest rate

$$R_t = 1 + \frac{\hat{\eta}_t}{\hat{\lambda}_t}. \quad (17)$$

3.2 Solution Method

We are interested in equilibria in which a precautionary money demand occurs. That means that we will work with equilibria where the cash-in-advance constraint becomes nonbinding in some states of the world. The method of parameterized expectations described in detail in Den Haan and Marcet (1990) will be applied. This technique is useful in our case as there is no additional difficulty when working with nonbinding constraints.

Our task is to solve the system (9)-(17) applying the numerical algorithm mentioned above. The method consists of approximating expectations in equilibrium equations by flexible functional forms. We parameterize two expectations, in the equations (10) and (11). In the original model we had one exogenous state variable A_t and one endogenous state variable k_t . However, after transforming the model, the growth rate of capital γ_t is only dependent on the technology shocks and therefore, it has no additional predictive power. We will approximate the expectations by second degree exponentiated polynomials that are a function of the technology shock A_t . To parameterize the expectations 'good' initial conditions are needed. We take as initial conditions for the parameters the ones that correspond to the results obtained from the log-linear approximation around steady state, a numerical method described in Uhlig (1995).

3.3 Calibration

We calibrate the model to match the quarterly US data. In our model the velocity is stationary, therefore, we will calibrate the model for a period in which the velocity does not exhibit increasing pattern, 1979:1-1998:10. For this particular period the average value of the income velocity is $\bar{v} = 6.95$, and the standard error $\sigma_v = 0.014$. We will set the discount factor to $\beta = 0.99$. Solow residuals are calculated using US quarterly data for real GDP, taking quarterly variations in the capital stock to be approximately zero, as in Cooley (1997). The correlation coefficient $\rho_A = 0.99$ and the standard error of technology shocks $\sigma_A = 0.008$. To achieve average velocity like in the data, we set the rate of depreciation of capital to $\delta = 0.08$. The steady state value of the technology \bar{A} is endogenously determined in order to reach the average growth rate of output per worker of 0.4% per period, i.e. $\bar{\gamma} = 1.004$. The average growth rate of money supply (M1) in the data is $\bar{\mu} = 1.013$. We will consider that the inverse of the intertemporal elasticity of substitution takes values in the interval $\theta \in [0.5, 2.5]$.

4 Discussion of the Equilibrium Behavior

4.1 Precautionary Money Demand

Money demand is given by equations (10) and (13) and it is a function of the money growth rate, the discount factor and the current technology shock.⁶ When the money growth rate increases, the steady state value of the nominal interest rate also increases, as can be seen in equation (12). This means that when μ increases, it is less attractive to hold money and once agents have money, it is more attractive to spend it on consumption. Increasing the money growth rate will result in a binding cash-in-advance constraint in more states of the world. On the other hand, decreasing the money growth rate will allow us to find nonbinding liquidity constraints in more periods.

According to our simulations, the precautionary money demand arises under different conditions for $\theta < 1$ and for $\theta > 1$. When the parameter θ is lower than unity, agents tend to make a precautionary money demand when a technology shock is high. The opposite holds when θ is higher than unity, that is, for a sufficiently high shock the cash-in-advance constraint tends to become binding.

The reaction of the money demand and the nominal interest rate to the technology shocks (for a given money growth rate) is plotted in Figure 1.⁷ In any period t for which the ratio $\frac{c_t}{m_t} < 1$, agents make a precautionary money demand. The first row of Figure 1 shows the evolution of the technology shocks, the second row shows the corresponding nominal interest rate, and the third one the evolution of the precautionary money demand. In the left and right columns of Figure 1 we plot the behavior for $\theta = 0.5$ and for $\theta = 2.5$. The two situations depicted in Figure 1 illustrate the general behavior we observe for $\theta < 1$ and $\theta > 1$, respectively.

Agents with high θ want to smooth their consumption path. When there is a positive technology shock, a direct way of isolating the effect of the shock on consumption is to make the cash-in-advance constraint binding, so that consumption is prevented from reacting too much to output fluctuations. When the money growth rate μ is sufficiently high, individuals never make a precautionary money demand because the opportunity cost of holding money is high, as argued in the first paragraph of this section. When $\theta > 1$, we find that the technology shock A_t is positively correlated with the nominal interest rate R_{t+1} (see the right column of Figure 1). This implies that the opportunity cost of holding money decreases in bad times. For low money growth rates agents might make a precautionary money demand when A_t is low. All unspent balances are the part of the current period wealth. They cannot be saved in the form of capital immediately because at the time when the t period financial exchange takes place, the decision on k_{t+1} has been already taken. But the balances unspent at time t can be saved as k_{t+2} . This will accelerate the capital accumulation which will imply an increase in output and consumption. In this way the fluctuations in consumption can be dampened.

When $\theta < 1$, agents care less about consumption smoothing. Because the nominal

⁶The growth rate of capital γ_t , the transformed marginal utility of wealth $\hat{\lambda}_t$ and the process of expectation formation are all functions of the realization of the technology shock in period t .

⁷The nominal interest rate is a function of the money growth rate, the discount factor and the technology shocks, as can be seen in equation (12).

interest rate R_{t+1} is negatively correlated with technology shock A_t (see the left column of Figure 1), it is more attractive to hold money in good times. Therefore the cash-in-advance constraint may become nonbinding when A_t is high.

4.2 Velocity Analysis

Income velocity of money v_t is the ratio between real output and real monetary balances. In terms of the transformed model we can write

$$v_t = \frac{A_t}{\hat{m}_t}. \quad (18)$$

We want to evaluate the changes in the income velocity of money and its volatility due to the fact that agents make a precautionary money demand. We will calculate the volatility of the money velocity as a standard error of the logarithm of the Hodrick-Prescott filtered velocity series.⁸ Our benchmark case will be an economy with the money growth rate that corresponds to the average level found in the data, $\mu^{data} = 1.013$. Lowering μ will lead to a precautionary money demand in more states of the world. We will therefore decrease the growth rate of money supply in order to observe how the behavior of the income velocity changes when nonbinding cash-in-advance constraints appear.

In the absence of uncertainty velocity does not depend on the money growth rate. However, in the presence of technology shocks money growth rate may influence the money demand and thus the value of the income velocity. To evaluate the correlation between the money growth rate and the income velocity we analyze first the behavior of real monetary balances and capital accumulation. We observe that when the cash-in-advance constraint is binding in all periods, or nonbinding in all periods, the average ratio between the real money demand and capital \hat{m}_t is negatively correlated with the growth rate of money supply. On the other hand, when the cash-in-advance constraint becomes nonbinding in some states of the world and binding in others, the average value of \hat{m}_t is positively correlated with μ . It is easier to understand what this behavior means when looking at the relationship between the growth rate of capital and the growth rate of money supply. When the cash-in-advance constraint is always binding or always nonbinding, average growth rate of capital is positively correlated with the growth rate of money supply. The opposite happens when agents make a precautionary money demand in some states of the world, average growth rate of capital is negatively correlated with μ . Thus we have a model with Tobin effect, that gets reversed in case when the cash-in-advance constraint becomes nonbinding in some states of the world. We must note that the average level of technology and money supply growth rate are not related. Taking into account the behavior of stationary real balances \hat{m}_t and the equation (18), we may derive the relationship between the income velocity and the money supply growth rate. Average velocity is positively correlated with μ when the cash-in-advance constraint remains binding in all periods, or it is nonbinding in all periods. Once the precautionary money demand appears in some

⁸Hodrick-Prescott filter is a standard procedure to eliminate trends. Even if the velocity in our model does not exhibit any trend, we apply the Hodrick-Prescott filter in order to compare the simulated and empirical results.

states of the world, the relationship is reversed, velocity is negatively correlated with the growth rate of money supply. Table 1 summarizes the properties of the model described above. We report in Table 1 the following variables for different values of relative risk aversion θ and different money growth rates μ : fraction of periods in which a precautionary money demand occurs, NONB, average growth rate of capital, $\bar{\gamma}$, average level of the ratio between real monetary balances and capital, \bar{m} , average income velocity of money, \bar{v} , and the volatility of the income velocity of money, σ_v .

With respect to the volatility of the income velocity we can see that in most cases it is decreasing with decreasing money growth rate. The presence of the precautionary money demand in some states of the world does not seem to affect much the fluctuations of the income velocity. In all analyzed cases volatility changes significantly when the cash-in-advance constraint becomes nonbinding in all periods. In fact it becomes independent of the parameter θ . In this case it is not costly to hold money. Money is used for all transactions but the cash-in-advance constraint does not restrict the consumption purchases. Observing the simulated results we must state that the velocity fluctuations are guided by fluctuations in the opportunity cost of holding money. We thus conclude that even if our model is a cash-only model, capital acts like credit good and the model can deliver reasonable velocity variability when the cash-in-advance constraint binds in all states of the world.

Comparing the simulated results with the US data, $\bar{v}^{data} = 6.95$ and $\sigma_v^{data} = 0.014$, we may say that the corresponding relative risk aversion parameter θ should be set to a value around 2-2.5.⁹ Taking into account the average money growth rate found in data, $\bar{\mu}^{data} = 1.013$, we see that it does not seem very likely that the individuals make a precautionary money demand in many periods, unless we face a period of a very restrictive monetary policy.

⁹Estimates for θ from various studies rather vary. For example, Canzoneri and Dellas (1998) propose a reasonable range of θ between 1 and 4.

5 Conclusion

We have used a simple stochastic growth model with money introduced via a cash-in-advance constraint to analyze the behavior of the income velocity of real balances. The analysis of the precautionary money demand shows that the result of Svensson (1985) holds in a more general framework. Compared to Svensson (1985), however, the presence of capital in the model reverses the relationship between the precautionary money demand and technology shocks.¹⁰ In the model of Svensson (1985) agents who want to smooth their consumption path might make a precautionary money demand in good times. By spending the extra balances in bad times they avoid high consumption fluctuations.¹¹ In the model with capital, agents keep the cash-in-advance constraint binding under a positive technology shock and thus prevent that consumption reacts too much to output fluctuations. Individuals might make a precautionary money demand in bad times. Unspent balances can be saved in the future period in the form of capital. Higher capital accumulation accelerates growth and the effects of low shocks on consumption can be thus slightly dampened.

We also find that in the model in which the inflation increases capital accumulation, when the cash-in-advance constraint is always binding or always nonbinding, the effect reverses when the precautionary money demand occurs in some states of the world. Concerning the volatility of the money velocity, the model can deliver empirically plausible velocity fluctuations when the cash-in-advance constraint always binds. That implies that in our economy capital acts like credit good in Hodrick, Kocherlakota and Lucas (1991). We find that in more practical terms the precautionary money demand will not be present in the economy unless we deal with an unusually contractive monetary policy.

¹⁰For more detailed description of the effects see Hromcová (2004).

¹¹Svensson (1985) develops the result assuming identically independently distributed technology shocks. Nevertheless, performing a numerical analysis we can show that the result remains unchanged for correlated technology shocks.

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θ	μ	NONB	$\bar{\gamma}$	\bar{m}	\bar{v}	σ_v
0.5	1.013	0	1.0022	0.00871	12.18	0.068
0.5	1.01	0	1.00217	0.00874	12.14	0.068
0.5	1.00	0.1	1.002	0.0089	11.88	0.067
0.5	0.999	0.2	1.00202	0.00888	11.887	0.067
0.5	0.998	0.3	1.00204	0.00886	11.85	0.066
0.5	0.997	0.42	1.00208	0.00883	11.82	0.064
0.5	0.996	0.58	1.00217	0.00873	11.74	0.059
0.5	0.9955	0.59	1.00217	0.00873	11.73	0.059
0.5	0.995	0.72	1.00234	0.00856	11.65	0.051
0.5	0.994	0.83	1.00253	0.00838	11.66	0.045
0.5	0.993	0.96	1.00289	0.00801	11.82	0.034
0.5	0.992	1	1.00282	0.00808	11.23	0.009
0.9	1.013	0	1.00275	0.00975	9.50	0.015
0.9	1.01	0	1.00272	0.00978	9.48	0.015
0.9	1.00	0	1.00261	0.00989	9.37	0.015
0.9	0.998	0	1.00259	0.00991	9.35	0.015
0.9	0.996	0	1.00157	0.00993	9.33	0.015
0.9	0.993	0	1.00254	0.00996	9.30	0.015
0.9	0.991	0.43	1.00261	0.00989	9.37	0.014
0.9	0.9908	0.65	1.00264	0.00986	9.39	0.014
0.9	0.9906	0.68	1.00264	0.00986	9.39	0.013
0.9	0.9904	0.71	1.00263	0.00987	9.38	0.013
0.9	0.9902	0.96	1.00248	0.0100	9.23	0.010
0.9	0.99	1	1.00238	0.0101	9.13	0.009

Table 1: Changes in the income velocity of money and its volatility due to the precautionary money demand:

θ – relative risk aversion,

μ – money growth rate,

NONB – fraction of periods in which a precautionary money demand arises,

$\bar{\gamma}$ – average growth rate of capital,

\bar{m} – average level of consumption to capital ratio,

\bar{v} – average level of the income velocity of money,

σ_v – volatility of the income velocity of money obtained from simulated series after HP filtering

θ	μ	NONB	$\bar{\gamma}$	\bar{m}	\bar{v}	σ_v
1.5	1.013	0	1.00338	0.01152	8.27	0.0074
1.5	1.01	0	1.00335	0.00155	8.25	0.0074
1.5	1.00	0	1.00326	0.01163	8.19	0.0074
1.5	0.998	0	1.00325	0.01165	8.18	0.0074
1.5	0.995	0	1.00324	0.01166	8.17	0.0074
1.5	0.993	0.002	1.00323	0.01167	8.16	0.0074
1.5	0.992	0.025	1.00323	0.01167	8.17	0.0073
1.5	0.991	0.23	1.00326	0.01164	8.18	0.0072
1.5	0.99	0.72	1.00334	0.01155	8.24	0.0070
1.5	0.989	1	1.00374	0.01116	8.49	0.009
1.5	0.988	1	1.00372	0.01118	8.48	0.009
1.5	0.987	1	1.0037	0.01119	8.47	0.009
2.0	1.013	0	1.00376	0.01314	7.47	0.0148
2.0	1.01	0	1.00373	0.01317	7.45	0.0148
2.0	1.00	0	1.00369	0.01321	7.43	0.0147
2.0	0.998	0	1.00368	0.01322	7.42	0.0147
2.0	0.995	0	1.00367	0.01323	7.42	0.0147
2.0	0.99	0.5	1.0037	0.01321	7.43	0.0147
2.0	0.989	0.78	1.00387	0.01303	7.52	0.0157
2.0	0.988	1	1.0047	0.01220	7.93	0.009
2.0	0.986	1	1.00467	0.01223	7.91	0.009
2.0	0.985	1	1.00465	0.01226	7.90	0.009
2.5	1.013	0	1.00404	0.01486	6.783	0.0184
2.5	1.01	0	1.00403	0.01488	6.774	0.0184
2.5	1.00	0	1.00398	0.01493	6.752	0.0183
2.5	0.998	0	1.00397	0.01494	6.748	0.0183
2.5	0.995	0	1.00396	0.01495	6.743	0.0182
2.5	0.993	0.004	1.00396	0.01495	6.741	0.0182
2.5	0.99	0.2	1.00396	0.01495	6.742	0.0182
2.5	0.989	0.56	1.00401	0.01490	6.764	0.0187
2.5	0.988	0.77	1.00421	0.01470	6.863	0.021
2.5	0.987	0.98	1.00567	0.01324	7.46	0.009
2.5	0.986	1	1.00565	0.01326	7.45	0.009

Table 1: Continued

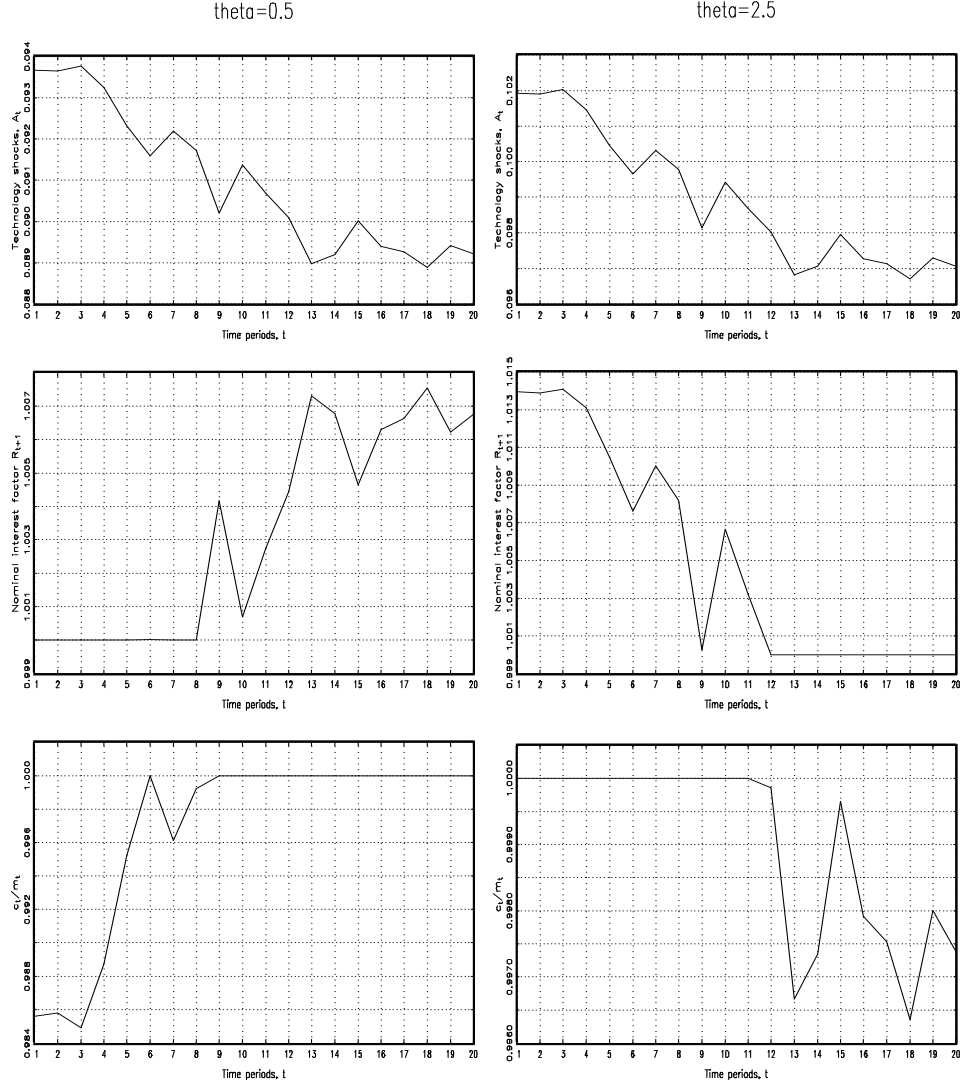


Figure 1: Relationship between technology shocks A_t , nominal interest rate R_{t+1} , and the precautionary money demand c_t/m_t , for $\beta = 0.99$, for the relative risk aversion lower and higher than unity and a given money growth rate:

- a) Left column: results for $\theta = 0.5$, $\mu = 0.997$,
- b) Right column: results for $\theta = 2.5$, $\mu = 0.989$.